Determination of the HQET Parameters from the $B \rightarrow X_s\gamma$ Decay

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Abstract

We combine the resummations for radiative corrections and for the heavy quark expansion to study the inclusive radiative decay $B \rightarrow X_s\gamma$. The infrared renormalon ambiguity is also taken into account. Including both theoretical and experimental uncertainties, we determine the allowed domain for the HQET parameters $\bar{\Lambda}$ and λ_1 centered at $\bar{\Lambda} = 0.65$ GeV and $\lambda_1 = -0.71$ GeV^2.

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With the progress of the heavy quark effective theory (HQET), we have gained better insight to the dynamics of heavy systems. For example, the inclusive semileptonic B-meson decay rates can be expanded in powers of $1/m_b$. To $O(1/m_b^2)$ (*ie* within an accuracy of about 5%), three parameters $\bar{\Lambda} = m_B - m_b$, $\lambda_1 = \langle (iD)^2 \rangle$ and $\lambda_2 = \langle \sigma \cdot G \rangle$ are relevant. Except for λ_2 which can be obtained directly from the B^*-B mass splitting, $\bar{\Lambda}$ and λ_1 are poorly determined. Recently, we have formulated the perturbative QCD (PQCD) approach to the inclusive semileptonic B-meson decays, which combines the resummation technique and the HQET based operator product expansion [1]. In this approach the charged lepton spectrum is expressed as the convolution of a hard subprocess with a jet function and a universal B-meson distribution function.

In this letter we shall determine the parameters $\bar{\Lambda}$ and λ_1, which are equivalent to the first and second moments of the B-meson distribution function, respectively, from the photon energy specrum of the radiative decay $B \to X_s \gamma$ [2]. It should be emphasized that the relation $\bar{\Lambda} = m_B - m_b$, and thus the extraction of the pole mass m_b of the b quark, suffer the infrared (IR) renormalon ambiguity of power $\Lambda_{\rm QCD}/m_b$ [3]. Hence, we regard $\bar{\Lambda}$ as being related to the first moment of the distribution function,

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instead of to the b quark mass. This viewpoint is satisfactory enough in the sense that the moments extracted here can be consistently employed to make predictions for other processes because of the universality of distribution functions. This makes possible a model-independent determination of the Cabibbo-Koboyashi-Maskawa (CKM) matrix elements $|V_{cb}|$ and $|V_{ub}|$ from the inclusive semileptonic decays.

On the other hand, the ambiguity of $O(\Lambda_{\mathrm{QCD}}/m_b)$ in the definition of the pole mass turns out to be cancelled by the ambiguity contained in loop corrections [3], when one evaluates the total decay widths of heavy mesons. In the factorization theorem formulated at the meson level, we adopt the B-meson mass m_B and thus avoid the ambiguity in m_b. It is then found that the IR renormalon ambiguity appearing in the perturbative resummation starts at $O(\Lambda_{\mathrm{QCD}}^2/m_b^2)$, consistent with the conclusion in [3]. This $O(\Lambda_{\mathrm{QCD}}^2/m_b^2)$ ambiguity will be taken into account as a source of theoretical uncertainties. After including the uncertainty from the experimental data, we determine $\bar{\Lambda} = 0.65^{+0.42}_{-0.30}$ GeV and $\lambda_1 = -0.71^{+0.70}_{-1.16}$ GeV^2.

The first two moments of the photon energy spectrum of the decay $B \rightarrow X_s\gamma$ have been computed using the expansions in $1/m_b$ and in α_s [4]. Without considering the errors of data, the parameter $\bar{\Lambda} \approx 450$ MeV

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was extracted. In our approach the PQCD expansion in α_s is resummed up to next-to-leading logarithms, and the nonperturbative heavy quark expansion in $1/m_b$ is resummed into the B-meson distribution function. Furthermore, both the IR renormalon ambiguity as a source of theoretical uncertainties and the errors of data as a source of experimental uncertainties are included. Therefore, our analysis is more complete. In [5] the contributions to the $B \rightarrow X_s\gamma$ decay from the $b \rightarrow sg$ transition through the $s \rightarrow \gamma$ and $g \rightarrow \gamma$ fragmentation functions were studied, and found to be negligible. This observation hints that we concentrate only on the $b \rightarrow s\gamma$ transition. Single logarithms in the fragmentation functions were summed using the renormalization-group (RG) method. In this work we employ the more sophiscated resummation technique to organize the double logarithms involved in the $b \rightarrow s\gamma$ decay.

The one-loop constraint on $\bar{\Lambda}$ and λ_1, $\bar{\Lambda} > [0.32 - 0.07(\lambda_1/0.1\text{GeV}^2)]$ GeV, has been obtained from the first moment of the invariant mass spectrum of the $B \rightarrow X_c l\nu$ decay using the same expansion in both $1/m_b$ and α_s [6]. This constraint, however, defines only an open domain. QCD sum rules are an alternative approach to the determination of the nonperturbative parameters, which give $\lambda_1 = -0.6 \pm 0.1$ GeV^2 [7]. It is also worthwhile to mention the

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lattice extractions of $\bar{m}_b(m_b) = 4.17 \pm 0.06$ GeV [8] and $\bar{m}_b(m_b) = 4.0 \pm 0.1$ GeV [9], which are close to the lower bound of our predictions. It is obvious that our predictions are not only concrete, but consistent with the conclusions in the literature.

The effective Hamiltonian for the process $b \to s\gamma$ written in terms of dimension-6 operators is

$$H_{\text{eff}}(b \to s\gamma) = -\frac{G_F}{\sqrt{2}}\lambda_t \sum_{j=1}^{8} C_j(\mu)\mathcal{O}_j(\mu) , \qquad (1)$$

with G_F the Fermi coupling constant, $C_j(\mu)$ the Wilson coefficients evaluated at the scale μ, and $\lambda_t = V_{tb}V_{ts}^*$ the product of the CKM matrix elements. The definition of the operators $\mathcal{O}_j$ is referred to [10]. Here we show only the relevant operator

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\bar{s}_\alpha\sigma^{\mu\nu}(m_b(\mu)R + m_s(\mu)L)b_\alpha F_{\mu\nu} , \qquad (2)$$

with $R = (1 + \gamma_5)/2$ and $L = (1 - \gamma_5)/2$. Since the mixing between the operators $\mathcal{O}_1, ..., \mathcal{O}_6$ and the operators $\mathcal{O}_7$ and $\mathcal{O}_8$ vanishes at one-loop level under the infinite renormalization, two-loop calculations of $O(eg_s^2)$ and $O(g_s^3)$ are necessary in order to obtain $C_7(\mu)$ and $C_8(\mu)$ in the leading logarithmic approximation, where e and g_s are the electromagnetic and strong couplings,

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respectively. Recently, the $O(\alpha_s)$ corrections to C_7 and C_8 have been computed in [11]. For the $b \rightarrow s\gamma$ transition, the contributions from all the operatorrs can be included by simply employing the effective Wilson coefficient $C_7^{\text{eff}} = C_7 + Q_d C_5 + 3 Q_d C_6$ [10], Q_d being the charge of the d quark.

The analysis in [10] involves only lowest-order PQCD corrections to the spectator model, and thus addresses nothing on the nonperturbative parameters. However, the explicit $O(\alpha_s)$ expressions for the decay width of $b \rightarrow s\gamma$ help to understand our formalism. The infrared single pole appearing in the one-loop calculation is absorbed into the distribution function, the double logarithms $\ln^2(m_s/m_b)$ with m_s the s quark mass, are absorbed into the jet function, and the single logarithms $\ln(\mu/m_b)$ are absorbed into the hard part [1]. Employing the resummation technique and the RG method, these large corrections are grouped into a Sudakov factor. This systemmatic summation of large logarithms has not been achieved in the literature. The distribution function can be regarded as the consequence of the resummation of the heavy quark expansion [12].

The factorization formula is then written as

$$\frac{1}{\Gamma^{(0)}} \frac{d\Gamma}{dE_\gamma} = m_B \int_x^1 dz \int b \, db \, f(z,b) J(z,x,m_B,b) H(x) \exp[-S(m_B,b)] , \quad (3)$$

with the tree-level decay rate $\Gamma^{(0)}$, the jet function J, the hard part H, and the Sudakov exponent S given by

$$\Gamma^{(0)} = \frac{m_B^5}{32\pi^4}|C_7^{\text{eff}}(m_B)G_F\lambda_t|^2\alpha_{\text{em}} \ , \tag{4}$$

$$J = J_0(\sqrt{z - x}m_Bb) \ , \quad H = x^2 \ , \tag{5}$$

$$S = 2\int_{1/b}^{m_B} \frac{dp}{p} \int_{1/b}^{p} \frac{d\mu}{\mu}A(\alpha_s(\mu)) - \frac{5}{3}\int_{1/b}^{m_B} \frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{\pi} \ , \tag{6}$$

$$A = \mathcal{C}_F\frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f\right]\left(\frac{\alpha_s}{\pi}\right)^2 \ , \tag{7}$$

with $\mathcal{C}_F = 4/3$ the color factor. The variable x is defined by $x = 2E_\gamma/m_B$, E_γ being the photon energy. Note that the second term of A is a two-loop result [13]. For consistency, the two-loop expression of $\alpha_s(\mu)$ is inserted into the integral of S. Note that the B-meson mass m_B, instead of the b-quark mass m_b, appears in the expression of $\Gamma^{(0)}$ as stated before. The choice of the scale m_B for C_7^{eff}, the same as the upper bound of the evolution of the Sudakov factor, follows the three-scale factorization theorem developed recently [14].

The argument z of the function $f(z,b)$ is the momentum fraction, and b is the conjugate variable of the transverse momentum carried by the b quark. Including these transverse degrees of freedom, we set m_s to zero, and let $1/b$ play the role of an infrared cutoff in the resummation for the jet function as shown in Eq. (6). Since the intrinsic b dependence (the perturbative b

dependence has been collected into the exponent S) is not known yet, we take the ansatz $f(z,b) = f(z) \exp(-\Sigma(b))$, which leads to $\Sigma \to 0$ as $b \to 0$ according to the definition $f(z, b = 0) \equiv f(z)$. It is also natrual to assume $\Sigma > 0$ for all b from the viewpoint that the b quark is bounded inside the B meson. Hence, the intrinsic b depedence provides further suppression. The nonperturbative function $f(z)$, identified as the B-meson distribution function, can be expressed as the matrix element of the b quark fields, whose first three moments are [12]

$$\int_0^1 f(z)dz = 1 \; ,$$
$$\int_0^1 f(z)(1-z)dz = \frac{\bar{\Lambda}}{m_B} + O(\Lambda_{\text{QCD}}^2/m_B^2) \; ,$$
$$\int_0^1 f(z)(1-z)^2 dz = \frac{1}{m_B^2}\left(\bar{\Lambda}^2 - \frac{\lambda_1}{3}\right) + O(\Lambda_{\text{QCD}}^3/m_B^3) \; . \qquad (8)$$

Though the exponent Σ is unknown, we can, however, extract its leading behavior by means of the IR renormalon analysis. Note that the perturbative Sudakov factor e^{-S} in Eq. (3) becomes unreliable as the transverse distance b approaches $1/\Lambda_{\text{QCD}}$. Near this end point, $\alpha_s(1/b)$ diverges, and IR renormalon contributions are significant. We reexpress the RG result of the evolution of the distribution function, which is contained in the second

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term of S, as

$$W = \exp\left[4\pi\mathcal{C}_F \int \frac{d^4l}{(2\pi)^4} \frac{v_\mu v_\nu}{(v\cdot l)^2} 2\pi\delta(l^2)\alpha_s(l_T^2)e^{i\mathbf{l}_T\cdot\mathbf{b}}N^{\mu\nu}\right] . \tag{9}$$

The loop integral corresponds to the correction from a real soft gluon attaching the two valence b quarks, whose propagators have been replaced by the eikonal lines in the direction $v = (1,1,\mathbf{0})$ [13]. The tensor $N^{\mu\nu} = g^{\mu\nu} - (n^\mu l^\nu + l^\mu n^\nu)/(n\cdot l) + n^2 l^\mu l^\nu/(n\cdot l)^2$ comes from the gluon propagator in axial gauge $n\cdot A = 0$. We have set the argument of the running α_s to l_T^2, which is conjugate to the scale b of the distribution function.

Substituting the identity $\alpha_s(l_T^2) = \pi \int_0^\infty d\sigma \exp[-2\sigma\beta_1 \ln(l_T/\Lambda_{\rm QCD})]$, $\beta_1 = (33-2n_f)/12$, into Eq. (9), and performing the loop integral for $n \propto (-1,1,\mathbf{0})$ [13], we obtain

$$W = \exp\left[\mathcal{C}_F \int_0^\infty d\sigma \left(\frac{b\Lambda_{\rm QCD}}{2}\right)^{2\sigma\beta_1} \frac{\Gamma(-\sigma\beta_1)}{\Gamma(1+\sigma\beta_1)}\right] . \tag{10}$$

It is easy to observe that the pole of $\Gamma(-\sigma\beta_1)$ at $\sigma \to 0$ gives the perturbative anomalous dimension of the distribution function appearing in Eq. (6). The extra poles at $\sigma \to 1/\beta_1$, $2/\beta_1$,..., then correspond to the IR renormalons, giving corrections of powers b^2, b^4,..., respectively. These renormalons generate singularities, which must be compensated by the nonperturbative power correctons in order to have a well-defined perturbative expansion. Note that

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the renormalon ambiguity starts with the power $(b\Lambda_{\rm QCD})^2$, instead of $b\Lambda_{\rm QCD}$ [3], because the function W does not include the self-energy corrections to the b quark, which vanish under the eikonal approximation.

Using the "minimal" ansatz [15], *ie* picking up only the leading (left-most) renormalon contribution, we parametrize the exponent $\Sigma(b)$ by $\Sigma(b) = c'm_B^2 b^2$, corresponding to the fact that the power corrections start at $O(b^2)$. Certainly, other parametrizations consistent with this fact are equally good. The IR renormalon ambiguity in other terms of S can be absorbed into $\Sigma(b)$. Combined with the Sudakov exponent, which is approximated by $S \approx 0.025 m_B^2 b^2$, Eq. (3) gives the differential branching ratio of the decay $B \to X_s\gamma$,

$$\frac{dBR}{dE_\gamma} = r\frac{x^2}{c}\int_x^1 dz f(z)e^{-(z-x)/c} \, , \tag{11}$$

with $c = 4c' + 0.1$ and $r = 2\Gamma^{(0)}\tau_B/m_B$. By varying the parameter c under the constraint $c > 0.1$, the theoretical uncertainty arising from our formalism is taken into account. Assuming the values $m_B = 5.279$ GeV, $G_F = 1.16639 \times 10^{-5}$ GeV^{-4} [16], $C_7^{\rm eff}(m_B) = -0.306 \pm 0.050$, $|\lambda_t| = 0.040 \pm 0.004$, $\alpha_{\rm em} = 1/130$ [10], and the B-meson lifetime $\tau_B = 1.60 \pm 0.03$ ps [17], we have $r = 1.90 \times 10^{-4}$ GeV^{-1} with about 50% uncertainty. We find that the

allowed values of c run from 0.1 to 0.2, *ie* c' associated with the nonperturbative exponent is a small number. It indicates that perturbative corrections are more important than nonperturbative corrections, consistent with the requirement from heavy quark symmetry.

Since two parameters are to be determined, the function $f(z)$ is modeled by the two-parameter form proposed in [1],

$$f(z) = N \frac{z(1-z)^2}{[(z-a)^2 + \epsilon z]^2} \, , \tag{12}$$

where N is the normalization constant, and the parameters a and ϵ will be adjusted to fit the CLEO data within errors. Note that the specific form of $f(z)$ is not essential. Any two-parameter wave function serves the same purpose. $\bar{\Lambda}$ and λ_1 are then obtained from the moments of $f(z)$ using Eq. (8). The numerical analysis proceeds in the following way: We choose the central value $r = 1.90 \times 10^{-4}$ GeV^{-1} first, and start with a value of $a \leq 1$. For an arbitrary value of ϵ, we vary c under the constraint $c > 0.1$. If there exists a finite range of c such that the predictions from Eq. (11) fall into the error bars of the CLEO data [2], this ϵ is acceptable. Repeating this procedure for different ϵ, an allowed range of ϵ is found. By changing a, we obtain the allowed domain for a and ϵ, from which the corresponding domain of $\bar{\Lambda}$ and

λ_1 is determined.

Results are shown in Fig. 1, in which the solid curves correspond to $a = 0.91,\ 0.92,...,$ and 1.00 from right to left, and a point on each curve corresponds to a value of ϵ. For $a \leq 0.90$, no acceptable ϵ exists. These curves form the corresponding allowed domain of $\bar{\Lambda}$ and λ_1. Including the errors of r, the domain enlarges by 20%. If there is no the constraint $c > 0.1$, the upper bound of $\bar{\Lambda}$ will increase 30%. The dashed curves are quoted from [6], with the left one and the right one obtained from the first and the second moments of the invariant mass spectrum of the $B \rightarrow X_c l \nu$ decay, respectively. The space above the dashed curves is the allowed domain in [6]. The third information from the ratio of partial widths $R_\tau = \Gamma(B \rightarrow X_c \tau \nu)/\Gamma(B \rightarrow X_c e \nu)$ introduces a constraint from the top. However, this constraint is not yet convincing [6], and still leaves the allowed domain an open one. Though the overlap of our results with those in [6] is not very large, it is reasonable to claim that they are consistent with each other, because the theoretical uncertainties of the approach in [6] was not estimated.

We take the middle of the $a = 0.95$ curve as the central values, which lead to $\bar{\Lambda} = 0.65^{+0.42}_{-0.30}$ GeV and $\lambda_1 = -0.71^{+0.70}_{-1.16}$ GeV^2. The bounds of these extractions are indeed very large. However, we emphasize that they are the

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consequence of the inclusion of as more as possible theoretical and experimental uncertainties into our formalism. These bounds will definitely be reduced when, for example, more accurate data are available. If the errors of the data become half of current ones, we shall obtain $\bar{\Lambda} = 0.65^{+0.34}_{-0.13}$ GeV and $\lambda_1 = -0.71^{+0.57}_{-1.06}$ GeV^2. As a simple estimation, we extract from $\bar{\Lambda}$ the b-quark mass $m_b = 4.63^{-0.42}_{+0.30}$ GeV, which certainly suffers the IR renormalon ambiguity. Employing the central values, we determine the B-meson distribution function

$$f(z) = \frac{0.02647z(1-z)^2}{[(z-0.95)^2 + 0.0034z]^2} \,, \tag{13}$$

which can be used to determine the inclusive semileptonic and nonleptonic decay spectra in the future.

To highlight the resummation effect, we present the predictions for the photon energy spectra of the $B \to X_s\gamma$ decay derived from the nonperturbative HQET distribution function alone, and from our formula including the perturbative resummation in Eq. (11). The expression of the former is simply $dBR/dE_\gamma = rx^2f(x)$. Substituting $r = 1.90$ and Eq. (13) into the above expression and into Eq. (11) with $c = 0.15$, we obtain the spectra shown in Fig. 2. The CLEO data are also displayed. It is obvious that the spectrum

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from $f(x)$ has a sharp peak near the high end of E_γ, which satisfies the HQET, but is in conflict with the data. The predictions match the data only after the suppression effect is included, which possess a softer profile. Note that a naive fitting without the suppression effect needs a board distribution function, leading to a value of m_b as small as 3.0 GeV.

The branching ratio can be evaluated simply by integrating Eq. (11) over E_γ. We obtain $BR = 2.80^{+0.14}_{-0.50} \times 10^{-4}$, where the central value corresponds to the solid curve in Fig. 2 ($c = 0.15$), and the upper bound and lower bound to $c = 0.13$ and $c = 0.25$, respectively. Our prediction is close to the standard-model estimation $(2.8 \pm 0.8) \times 10^{-4}$ [18], and to the CLEO data $(2.75 \pm 0.67) \times 10^{-4}$ from the B-reconstruction analysis [2].

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Figure Captions

Fig.1: The allowed domain for $\bar{\Lambda}$ and λ_1. The dashed curves are the constraints quoted from [6].

Fig.2: The photon energy spectra derived from the B-meson distribution function alone (dashed curve), and from the inclusion of the suppression effect (solid line). The CLEO data are also shown [2].